Extension of the Snow Fun Ski Resort Problem

Suppose that the operating cost of the snowmaking equipment is actually 30% higher than the owner had estimated, that is, $13,000 if the snowfall is heavy, $65,000 if it is moderate, and $117,000 if it is light.

How will the increased operating cost affect the owner’s optimal decision?

As shown above, the best decision now is to let the larger hotel run the ski resort and the profit is $45,000

Calculate the EVPI, and determine that maximum amount that could be paid for a perfect forecast.

If the snowfall is more than 40”, the owner should operate the resort without a snowmaker (profit=$120,000, probability=40%). If the snowfall is between 20 to 40 inches or if it is less than 20 inches, the owner should let the larger hotel run the resort (profit=$45,000, probability=20%+40%=60%).

\[
EVPI = $120,000 \times 0.40 + $45,000 \times 0.60 = $75,000
\]

Since the optimal profit without perfect information is $45,000, the maximum amount the owner can pay for perfect information is $75,000 - $45,000 = $30,000
Decision Tree Final Problem

The NC Airport Authority is trying to solve a difficult problem with the over-crowded Raleigh-Durham airport. There are three options to consider:

A. The airport could be totally redesigned and rebuilt at a cost of $8.2 million. The present value of increased revenue from a new airport is in question. There is a 70% chance this present value would be $11 million, a 20% chance present value would be $5 million, and a 10% chance present value would be $1 million, depending on whether the airport is a success, moderate success, or a failure.

B. The airport could be remodeled with a new runway for a cost of $4.7 million. The present value of increased revenue would be $6 million (80% chance) or $3 million (20% chance).

C. They could do nothing with the airport and suffer a loss of revenue of either $1 million (65% chance) or $4 million (35% chance).

1. Construct a decision tree to help the Airport Authority so as to maximize the present value of profit.

2. How much would we be willing to pay for perfect information about the success of a brand new airport?

3. How much would we be willing to pay for perfect information about the success of a remodeled airport?

Solution to 1: The best option is to remodel since it maximizes expected profit 0.7M

Solution to 2: With perfect information on redesign, it should be taken only if there is total success, else we should go with remodeling. EVPI=.7*2.8+.3*.7=2.17. Value=2.17-.7=1.47M

Solution to 3: With perfect information on remodeling, it should be taken only if there is success, else we should go with redesign. EVPI=.8*1.3+.2*.6=1.16. Value=1.16-.7=0.46M
The XYZ company owns and operates a 3-seater airplane to show tourists the Great Barrier Reef in Cairns, Australia. The company uses a reservation system, wherein tourists call in advance and make a reservation for aerial viewing the following day. Unfortunately, often passengers holding a reservation might not show up for their flight. Assume that the probability of a passenger not showing up for a flight is 15%.

a) Show the probability distribution of the number of passengers that will show up

<table>
<thead>
<tr>
<th>No. Passengers</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>0---------------</td>
<td>0.003375</td>
</tr>
<tr>
<td>1---------------</td>
<td>0.057375</td>
</tr>
<tr>
<td>2---------------</td>
<td>0.325125</td>
</tr>
<tr>
<td>3---------------</td>
<td>0.614125</td>
</tr>
</tbody>
</table>

Remember, probabilities in your probability distribution should add up to 1

b) From your probability distribution, compute the probability that two or fewer passengers will show up

0.385875

c) From your probability distribution, compute the probability that at least 1 passenger will show up

0.996625

d) Find the expected value of the number of passengers that will show up

2.55

e) Find the standard deviation of the number of passengers that will show up

0.618466
Example problems from Chapter 5

Problem #1: Objectives

- To demonstrate the calculation of expected value and variance / standard deviation of a discrete random variable

Problem #1: Background

- A mining company plans to develop two potential gaussite reserves. Each reserve has a 30% probability of successfully yielding usable gaussite, and the success of each reserve is independent of the other. If either of the two reserves is successful, it will generate $4 million in profit; if both are successful, profits will be $7 million because excess supply will lower prices. If neither is successful, profits will be 0. Find the expected value and standard deviation of profit.

Expected Value and Variance Calculations

<table>
<thead>
<tr>
<th>Site 1</th>
<th>Site 2</th>
<th>Prob.</th>
<th>Rev.</th>
<th>E(X)</th>
<th>Var(X)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Success</td>
<td>Success</td>
<td>.3X.3=.09</td>
<td>$7 mill.</td>
<td>7X.09=.63</td>
<td>(7-2.31)^2 X.09 = 1.979649</td>
</tr>
<tr>
<td>Success</td>
<td>Failure</td>
<td>.3X.7=.21</td>
<td>$4 mill.</td>
<td>4X.21=.84</td>
<td>(4-2.31)^2 X.21 = .599781</td>
</tr>
<tr>
<td>Failure</td>
<td>Success</td>
<td>.7X.3=.21</td>
<td>$4 mill.</td>
<td>4X.21=.84</td>
<td>(4-2.31)^2 X.21 = .599781</td>
</tr>
<tr>
<td>Failure</td>
<td>Failure</td>
<td>.7X.7=.49</td>
<td>$0 mill.</td>
<td>0X.49=0</td>
<td>(0-2.31)^2 X.49 = 2.614689</td>
</tr>
</tbody>
</table>

1.00     2.31M    5.7939

Expected value and variance

- Expected value \( E(X)=\$2.31 \) Million
- Variance = 5.7939
- Standard deviation is the square root of variance; in our case std. dev. is \$2.41 \) Million
- Comment: Standard deviation is greater than the expected value (i.e. CV will be greater than 100%); thus, the project is very risky
Problem #2: Objectives & Background

Objective
• Calculating expected values using a joint probability distribution table

Background
• The YouDee Café serves the exotic Bernoulli Salmon at lunch and dinner. The number of customers ordering the salmon at lunch and dinner are given by the joint probability distribution shown on the next slide.
• The chef orders three fish each day at a cost of $3.50 per serving. Any fish left over at the end of the day is discarded.

Joint probability distribution

<table>
<thead>
<tr>
<th></th>
<th>Lunch</th>
<th>Dinner</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>.06</td>
<td>.10</td>
</tr>
<tr>
<td>1</td>
<td>.12</td>
<td>.20</td>
</tr>
<tr>
<td>2</td>
<td>.12</td>
<td>.20</td>
</tr>
<tr>
<td>Totals</td>
<td>.30</td>
<td>.50</td>
</tr>
</tbody>
</table>

Expected lunch and dinner demands

• What is the expected lunch demand?
  ▪ Prob. of lunch demand=0 is .3
  ▪ Prob. of lunch demand=1 is .5
  ▪ Prob. of lunch demand=2 is .2
  ▪ E(Lunch Demand)=0X.3+1X.5+2X.2=.9

• What is the expected dinner demand?
  ▪ Prob. of dinner demand=0 is .2
  ▪ Prob. of dinner demand=1 is .4
  ▪ Prob. of dinner demand=2 is .4
  ▪ E(Dinner Demand)=0X.2+1X.4+2X.4=1.2

Expected total demand and the probability of a stock out

• What is the expected total demand?
  ▪ E(Total Demand) = E(Lunch Demand) + E(Dinner Demand) = 0.9+1.2 = 2.1

• What is the probability of a stock out?
  ▪ Since the chef orders 3 fish each day, a stock out would occur when both lunch and dinner demand is for 2 fish; from the table, this probability is 0.08
Are lunch and dinner demands independent?

• Are lunch and dinner demands independent?
  ▪ Yes
  ▪ For example, the marginal probability of lunch demand=0 is .3
  ▪ The marginal probability of dinner demand=0 is 0.2
  ▪ The joint probability $P(\text{lunch and dinner demand}=0)$ is $0.06 (.3 \times 0.2 = 0.06)$
  ▪ Therefore lunch and dinner demands are independent

Break-even selling price

• What is the breakeven selling price (i.e. the price at which the expected revenue from sales of fish equals the cost of fish ordered)? Assume that a customer who would have ordered the fish but finds it sold out simply leaves rather than order something else.
  ▪ Hint: Expected revenue = price $\times$ number of fish sold.
  ▪ But, what is the (expected) number of fish sold at YouDee café?

Calculating the expected number of units sold

• Minimum total fish sold on any given day is 0
• Maximum total fish sold on any given day is 3
• On any given day, YouDee sells between 0 and 3 fish (i.e. possible values are 0, 1, 2 and 3)
  ▪ From the table, the probability of 0 fish being sold is 0.06
  ▪ Probability of 1 fish being sold is $0.12 + 0.04 + 0.2 = 0.36$
  ▪ Probability of 2 fish being sold is $0.12 + 0.04 + 0.2 = 0.36$
  ▪ Probability of 3 fish being sold is $0.08 + 0.2 + 0.08 = 0.36$  
  ▪ Expected number of fish sold
    ▪ $= 0 \times 0.06 + 1 \times 0.22 + 2 \times 0.36 + 3 \times 0.36 = 2.02$

Calculating the break-even selling price

• Cost per day = 3 fish $\times$ $3.50 = $10.50
• To break even, the $2.02$ fish that YouDee sells must earn a revenue of at least $10.50
  ▪ Break even selling price per fish should thus be $\frac{10.50}{2.02} = $5.20$