Extension of the Snow Fun Ski Resort Problem

Suppose that the operating cost of the snowmaking equipment is actually 30% higher than the owner had estimated, that is, $13,000 if the snowfall is heavy, $65,000 if it is moderate, and $117,000 if it is light.

How will the increased operating cost affect the owner’s optimal decision?

Calculate the EVPI, and determine that maximum amount that could be paid for a perfect forecast.
Another Decision Tree Problem

The NC Airport Authority is trying to solve a difficult problem with the over-crowded Raleigh-Durham airport. There are three options to consider:

A. The airport could be totally redesigned and rebuilt at a cost of $8.2 million. The present value of increased revenue from a new airport is in question. There is a 70% chance this present value would be $11 million, a 20% chance present value would be $5 million, and a 10% chance present value would be $1 million, depending on whether the airport is a success, moderate success, or a failure.

B. The airport could be remodeled with a new runway for a cost of $4.7 million. The present value of increased revenue would be $6 million (80% chance) or $3 million (20% chance).

C. They could do nothing with the airport and suffer a loss of revenue of either $1 million (65% chance) or $4 million (35% chance).

1. Construct a decision tree to help the Airport Authority so as to maximize profit (please show the profit/loss at each branch).

2. How much would we be willing to pay for perfect information about the success of a brand new airport?

3. How much would we be willing to pay for perfect information about the success of a remodeled airport?
The XYZ company owns and operates a 3-seater airplane to show tourists the Great Barrier Reef in Cairns, Australia. The company uses a reservation system, wherein tourists call in advance and make a reservation for aerial viewing the following day. Unfortunately, often passengers holding a reservation might not show up for their flight. Assume that the probability of a passenger not showing up for a flight is 15%.

a) Show the probability distribution of the number of passengers that will show up – *probabilities can be computed correctly only if you first list all possible outcomes, i.e. identify the sample space.*

<table>
<thead>
<tr>
<th>No. Passengers</th>
<th>Probability</th>
</tr>
</thead>
</table>

b) From your probability distribution, compute the probability that two or fewer passengers will show up

c) From your probability distribution, compute the probability that at least 1 passenger will show up

d) What is the expected value of the number of passengers that will show up?

e) Find the standard deviation of the number of passengers that will show up
Problem #1: Objectives

- To demonstrate the calculation of expected value and variance / standard deviation of a discrete random variable

Example problems from Chapter 5

Problem #1: Background

- A mining company plans to develop two potential gaussite reserves. Each reserve has a 30% probability of successfully yielding usable gaussite, and the success of each reserve is statistically independent of the other. If either of the two reserves is successful, it will generate $4 million in profit; if both are successful, profits will be $7 million because excess supply will lower prices. If neither is successful, profits will be 0. Find the expected value and standard deviation of profit (use the next slide to first list the 4 possible outcomes and then complete the remaining columns)

### Expected Value and Variance Calculations

<table>
<thead>
<tr>
<th>Site 1</th>
<th>Site 2</th>
<th>Prob.</th>
<th>Rev.</th>
<th>E(X)</th>
<th>Var(X)</th>
</tr>
</thead>
<tbody>
<tr>
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Expected value and variance

- Expected value $E(X)=?$
- Variance = ?
- Standard deviation is the square root of variance; in our case std. dev. is ?
Problem #2: Objectives & Background

**Objective**
- Calculating expected values using a joint probability distribution table

**Background**
- The YouDee Café serves the exotic Bernoulli Salmon at lunch and dinner. The number of customers ordering the salmon at lunch and dinner are given by the joint probability distribution shown on the next slide.
- The chef orders three fish each day at a cost of $3.50 per serving. Any fish left over at the end of the day is discarded.

### Joint probability distribution

<table>
<thead>
<tr>
<th>Dinner</th>
<th>Lunch</th>
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<tbody>
<tr>
<td></td>
<td>0</td>
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<tr>
<td>0</td>
<td>.06</td>
</tr>
<tr>
<td>1</td>
<td>.12</td>
</tr>
<tr>
<td>2</td>
<td>.12</td>
</tr>
<tr>
<td>Totals</td>
<td>.30</td>
</tr>
</tbody>
</table>

Expected lunch and dinner demands

- What is the expected lunch demand?
- What is the expected dinner demand?

Expected total demand and the probability of a stock out

- What is the expected total demand? [Hint: It is the sum of expected lunch and expected dinner demands]
- What is the probability of a stock out?
Are lunch and dinner demands independent?

- Are lunch and dinner demands independent?

Break-even selling price

- What is the break-even selling price (i.e. the price at which the expected revenue from sales of fish equals the cost of fish ordered)? Assume that a customer who would have ordered the fish but finds it sold out simply leaves rather than order something else.

  - Hint: Expected revenue = price \times number of fish sold.

  - But, what is the (expected) number of fish sold at YouDee café?

Calculating the expected number of units sold

- Minimum total fish sold on any given day is 0

- Maximum total fish sold on any given day is 3

- On any given day, YouDee sells between 0 and 3 fish (i.e. possible values are 0, 1, 2 and 3)
  - From the table, the probability of 0 fish being sold is 0.06
  - Probability of 1 fish being sold is ?
  - Probability of 2 fish being sold is ?
  - Probability of 3 fish being sold is ?

- Expected number of fish sold = ?

Calculating the break-even selling price

- Cost per day = 3 fish \times $3.50 = $10.50

- To break even, the (expected) number of fish that YouDee sells must earn a revenue of at least $10.50

- Break even selling price per fish should thus be ?